

Power System Stability Enhancement with STATCOM Power Oscillation Damping Controller

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Abstract- The paper presents design and analysis of STATCOM power oscillation damping controller. The Phillips-Heffron model of the Single Machine Infinite Bus power system installed with STATCOM has been derived and the systematic approach for designing STATCOM power oscillation damping controller has been presented, the controller places the Eigen value corresponding to mode of oscillation at desired location so that the system has desired degree of stability. The performance of controller has been examined at different system conditions, under different line loadings and the effectiveness of proposed controller is verified through MATLAB simulation.

Keywords: FACTS, STATCOM, Phillips-Heffron model, Power Oscillation Damping controller.

I. INTRODUCTION

Today's Power system is a complex network, consist of thousands of buses and hundreds of generators. The available power generation usually does not situated near load center, in order to meet the growing power demands; utilities have an interest in better utilization of available power system capacities, existing generation and existing power transmission network, instead of building new transmission lines and expanding substations. On the other hand, power flows in some of the transmission lines are overloaded, which has as an overall effect of deteriorating voltage profiles and decreasing system stability and security. In addition, existing traditional transmission facilities, in most cases, are not designed to handle the control requirements of complex and highly interconnected power systems. This overall situation requires the review of traditional transmission methods and practices, and the creation of new concepts, which would allow the use of existing generation and transmission lines up to their full capabilities without reduction in system stability and security. In the past, power systems could not be controlled fast enough to handle dynamic system condition. This problem was solved by over-design; transmission systems were designed with generous stability margins to recover from anticipated operating contingencies caused by faults, line and generator outages, and equipment failures.

Series capacitor, shunt capacitor, phase shifter are different approaches to increase the power system transmission lines load ability. In past days, all these devices are controlled and switched mechanically and were, therefore, relatively slow. They are very useful in a steady state operation of power systems but from a dynamical point of view, their time response is too slow to effectively damp transient oscillations. If mechanically controlled systems were made to respond faster, power system security would be significantly improved, allowing the full utilization of system capability while maintaining adequate levels of stability. This concept and advances in the field of power electronics led to a new approach introduced by the Electric Power Research Institute (EPRI) in the late 1980, called Flexible AC Transmission Systems (FACTS), it was answer to call for a more efficient use of already existing resources in present power systems while maintaining and even improving power system security and stability [1].

The development of interconnection of large electric power systems led to presence of spontaneous system oscillations at very low frequencies of order of 0.2-3.0 Hz. Once started, the oscillation would continue for a while and then disappear, or continue to grow, causing system separation and stability related problem [3]. In order to damp these power system oscillations and to increase power system stability, the Power System Stabilizer (PSS) have been used for many years, to date; a number of major electric power system plants in many countries are equipped with PSS [4]. However, PSSs suffer a drawback of being liable to cause great variations in the voltage profile and they may even result in leading power factor operation and losing system stability under severe disturbances. In addition, in a deregulated environment, placement may be problematical due to generator ownership recently appeared FACTS-based stabilizer offer an alternative way in damping power system oscillation. Although, the power oscillation damping duty of FACTS controllers often is not their primary function, the capability of FACTS based stabilizers to increase power system oscillation damping characteristics has been recognized [5]. STATCOM can improve power oscillation damping effectively, the power oscillation damping capability of STATCOM is required to be

investigated thoroughly for proper on line applications in changing operating conditions. Different approaches based on modern control theory have been applied to STATCOM based POD controller design. H. F. Wang [6] presented a modified linearized Phillips-Heffron model of a power system installed with STATCOM and addressed basic issues pertaining to design of STATCOM based power oscillation damping controller.

II. POWER SYSTEM INSTALLED WITH STATCOM

Figure 1, shows a single machine infinite bus power system installed with STATCOM connected through a transformer. The single-machine infinite-bus (SMIB) system used in this study is for better understanding of power oscillation damping and hence enhancement of system stability. An STATCOM based on pulse width modulation (PWM) technique is being considered, it consist of a coupling transformer, a VSC, and a dc energy storage device, the energy storage device is a relatively small dc capacitor, hence the STATCOM is capable of only reactive power exchange with the transmission system. If a dc storage battery or other dc voltage source were used to replace the dc capacitor, the controller can exchange real and reactive power with the transmission system. The STATCOM's output voltage magnitude and phase angle can be varied by changing the modulation index m and the phase angle ϕ of the operation of the converter switches, thus controlling the magnitude of the fundamental component of the converter ac output voltage.

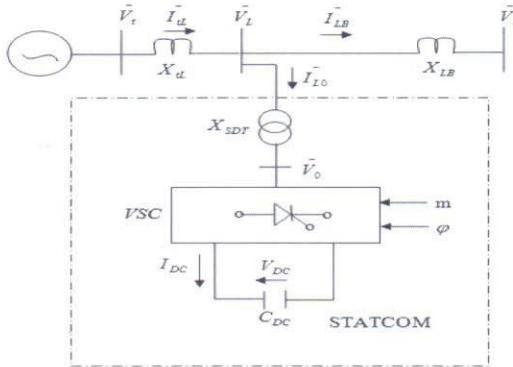


Fig. 1 STATCOM in SMIB power system.

The STATCOM is modeled as a voltage-sourced converter behind a Transformer as shown in Fig. 1 the STATCOM generates a controllable AC-voltage behind the leakage reactance.

$$\bar{V}_o = c V_{DC} (\cos \phi + j \sin \phi) = c V_{DC} \angle \phi$$

$$\frac{dV_{DC}}{dt} = \frac{I_{DC}}{C_{DC}} = \frac{c}{C_{DC}} (I_{Lod} \cos \phi + j I_{Lq} \sin \phi) \quad (1)$$

$c = mk$,

K is ratio between A C & D C Voltage & m = modulation index defined by the PWM. From fig.1

$$\bar{I}_{LB} = I_{tL} \bar{I}_{Lo}$$

where

$$I_{Lo} = \frac{V_L}{X_{SDT}} \quad (2)$$

$$\bar{I}_{LB} = \bar{I}_{tL} \frac{\bar{V}_L}{J X_{SDT}} \frac{\bar{V}_o}{J X_{SDT}}$$

$$V_L = V_t - J X_{tL} I_{tL}$$

we get

$$\bar{I}_{LB} = \bar{I}_{tL} \frac{\bar{V}_t - J X_{tL} I_{tL} - \bar{V}_o}{J X_{SDT}} \quad (3)$$

$$\bar{V}_t = J X_{tL} I_{tL} + J X_{LB} I_{LB} + \bar{V}_B \quad (4)$$

Substituting equation (3) into equation (4) which gives

$$\begin{aligned} \bar{V}_t &= J X_{tL} I_{tL} + J X_{LB} \left\{ \bar{I}_{tL} - \frac{\bar{V}_t - J X_{tL} I_{tL} - \bar{V}_o}{J X_{SDT}} \right\} + \bar{V}_B \\ &= J X_{tL} I_{tL} + \frac{J X_{tL} \cdot J X_{LB} I_{tL}}{J X_{SDT}} + J X_{LB} I_{LB} - J X_{LB} \left(\frac{\bar{V}_t - \bar{V}_o}{J X_{SDT}} \right) + \bar{V}_B \\ V_t &= J \left(x_{tL} + x_{tL} \cdot \frac{X_{LB}}{X_{SDT}} + X_{LB} \right) I_{tL} - J X_{LB} \frac{V_t}{J X_{SDT}} + \frac{J X_{LB} V_o}{J X_{SDT}} + V_B \\ \left(1 + \frac{X_{LB}}{X_{SDT}} \right) \bar{V}_t - \frac{X_{LB}}{X_{SDT}} V_o - \bar{V}_B &= J \bar{I}_{tL} \left\{ X_{tL} + X_{tL} \cdot \frac{X_{LB}}{X_{SDT}} + X_{LB} \right\} \end{aligned}$$

$$I_{tL} = I_{tLd} - j I_{tLq}$$

Following linearized state-space model of single machine infinite bus system installed with STATCOM is obtained as:

$$\dot{X} = A X + B U$$

$$\dot{X} = \begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{E}_q \\ \Delta \dot{E}_{fd} \\ \Delta \dot{V}_{DC} \end{bmatrix}, \quad U = \begin{bmatrix} \Delta C \\ \Delta \phi \end{bmatrix}$$

Where, ΔC and $\Delta \phi$ are the linearizations of the input control signals of the STATCOM

$$A = \begin{bmatrix} 0 & \omega_b & 0 & 0 & 0 \\ -\frac{k_1}{M} & -\frac{D}{M} & -\frac{k_2}{M} & 0 & -\frac{k_{PDC}}{M} \\ \frac{-k_4}{T_{do}'} & 0 & -\frac{k_2}{T_{do}'} & \frac{1}{T_{do}'} & -\frac{K_{qDC}}{T_{do}'} \\ -\frac{k_A k_5}{T_A} & 0 & -\frac{k_A k_6}{T_A} & -\frac{1}{T_A} & -\frac{k_A k_{VDC}}{T_A} \\ k_4 & 0 & k_8 & 0 & k_9 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -\frac{k_{pc}}{M} & \frac{k_{p\varphi}}{M} \\ -\frac{k_{qc}}{T_{do}} & -\frac{k_{q\varphi}}{T_{do}} \\ -\frac{k_A k_{vc}}{T_A} & -\frac{k_A k_{v\varphi}}{T_A} \\ k_{dc} & k_{d\varphi} \end{bmatrix}$$

ΔC = Deviation impulse width modulation index 'm' of the shunt inverter. By controlling m, the output voltage of the shunt converter is controlled.

$\Delta\varphi$ = Deviation in phase angle of the shunt converter voltage

The linearized dynamic model of above state model is shown by Fig. 2, where Δu being ΔC and $\Delta\varphi$.

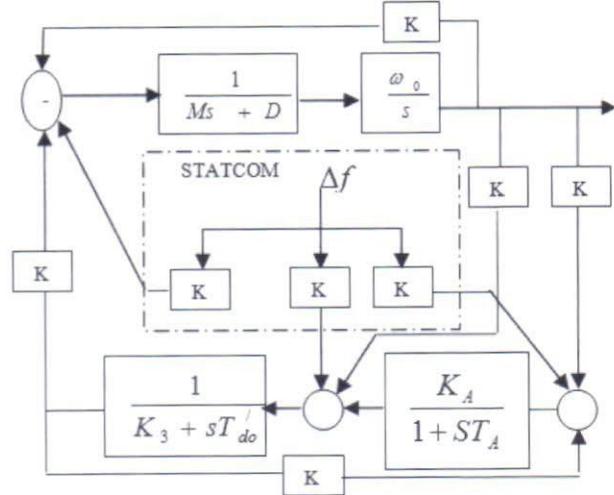


Fig.2 Phillips-Heffron model of power system installed with STATCOM

III. POD CONTROLLER

The dynamic characteristics of system can be influenced by location of eigenvalues, for a good system response in terms of overshoots /undershoot and settling time, a particular location for system eigenvalues is desired depending upon the operating conditions of the system. The damping power and the synchronizing power are related respectively, to real part and imaginary part of eigenvalue that correspond to incremental change in the deviation of the rotor speed and deviation of rotor angle[8], this Eigenvalue is known as electromechanical mode. Power oscillation damping can be improved if real part of eigenvalue associated with mode of oscillation can be shifted to left-side in complex s-plane as desired. This thesis present controller such that the closed loop designed system will have a desired degree of stability [9], and [10]. For the power system representation in state – space form, a closed loop gain matrix A-BK obtained by choosing the gain matrix K through state feedback control law $U = -KX$ will have all its eigenvalue lies in left side of complexes-plane.

It is an easy task to design power oscillation damping controller. Making use of proposed controller design approach, STATCOM based power oscillation damping controller is designed to damping of low frequency power oscillations. This has been attempted on a sample system. The expectation from STATCOM based POD controller is to provide instantaneous solution to power oscillation damping, the settling time as obtained from response of system is expected to be as small as possible. For minimizing settling time real part of eigenvalue corresponding to mode of oscillation are required to be shifted more and more on LHS of complex plane, this will require control effort. There is a hardware limit of any designed controller, for the case of STATCOM, in view of this, the control input parameters m and φ should be within their limit and the voltage of the DC link capacitor V_{dc} should be kept constant.

IV. DESIGN OF POD CONTROLLER

The Linearized state – space model of SMIB power system is obtained by phillips-heffron model as expressed by:

$$\dot{X} = AX + BU \quad (12)$$

Where A and B are the matrices of the system and input respectively. X is the system state vector, and U is the input state-vector. The matrices A and B are constant under the assumption of system linearity. If we use state feedback, that is, if we set $U = -KX$ where K is the chosen gain matrix, the equation becomes:

$$\dot{X} = (A - BK)X \quad (13)$$

And the problem is to allocate any set of eigenvalues to closed loop matrix $(A - BK)$ by choosing the gain matrix K. Here in this thesis the gain matrix K is chosen by MATLAB tool. The syntax is given below:

$$K = \text{place}(A, B, p) \quad (14)$$

Where vector p of desired self-conjugate closed-loop pole locations, place computes a gain matrix K such that the state feedback places the closed-loop poles at the locations p. In other words, the eigenvalues of $(A - BK)$ match the entries of p (up to the ordering). $K = \text{place}(A, B, p)$ computes a feedback gain matrix K that achieves the desired closed-loop pole locations p, assuming all the inputs of the plant are control inputs.

V. ANALYSIS OF CONTROLLER AND SIMULATION RESULTS

The effectiveness of proposed STATCOM POD controller for damping local mode oscillations has been demonstrated with SMIB system. The linearized state space model for SMIB installed with STATCOM is given above. Using pole-placement controller design technique STATCOM POD controller for SMIB has been designed. Also, to have improved damping and hence the small settling time of response, it is desired to shift the real part of eigenvalue corresponding to mode of oscillation to LHS in complex s-plane. To achieve this, it is desired to place eigenvalue corresponding to mode of oscillation at location on LHS of complex plane. The change in operating conditions of power system is common phenomenon, e.g., line loading varies over

a wide range of line may change, sometimes. For a good design of damping controller, besides the maximum effectiveness of the controller, the robustness of damping controller to the variations of power system operating conditions is an equally important factor to be taken under consideration. Hence, it is desirable for STATCOM POD controller that it must be able to respond for changes in operating point along with satisfactory performance. Therefore, it is extremely important to investigate the effect of load variations on the performance of the designed controller. In view of this, the performance of STATCOM POD controller at following operating conditions are studied (i) 20 percent decrease in line loading (ii) 20 percent increase in line loading. For performing such investigations, the eigenvalue analysis with simulation results at each loading condition has been done. The effect of variation in modulation index, and converter angle of converter are also considered on the performance of designed proposed controller. For performing such investigation again the eigenvalue analysis with simulation results in variation in modulation index and converter angle have been done.

- **Eigen- values analysis under weak power System and various line loadings**

Load → Controller ↓	Load decreased 20% (0.8) p.u.	Normal load (1.0) p.u.	Load increased 20% (1.2) p.u.
Without STATCOM	-98.6815	-98.6747	-98.6701
	0.0173 +	0.0184 +	0.0183 +
	8.0820i	8.9851i	9.8053i
	0.0173 -	0.0184 -	0.0183 -
	8.0820i	8.9851i	9.8053i
	-1.7055	-1.7146	-1.7189
With STATCOM	-98.8545	-98.8533	-98.8525
	-0.0270+	-0.0217+	-0.0181+
	5.0788i	5.6963i	6.2519i
	-0.0270 -	-0.0217 -	-0.0181 -
	5.0788i (0.00532)	5.6963i (0.00381)	6.2519i
	-1.3744	-1.4118	(0.0029)
With POD Controller	-0.1135	-0.0879	-1.4360 -0.0718
	-98.8545	-98.8533	-98.8525
	-0.1352+	-0.1086+	-0.0905+
	5.0788i	5.6963i	6.2519i
	-0.1352 -	-0.1086 -	-0.0905 -
	5.0788i (0.0266)	5.6963i	6.2519i
	-0.1135	(0.0191)	(0.0145)
	-1.3744	-0.0879	-0.0718
		-1.4118	-1.4360

Table1: Eigen values with STATCOM POD controller for weak SMIB system.

- **Eigen- values analysis under strong power system and various line loadings.**

The bolded row of this table represents the electromechanical mode eigenvalue and its damping ratio. It can be observed from the table that STATCOM with proposed controller greatly improve the system stability. Comparative study of table1 and table 2.

Load → Controller ↓	Load decreased 20% (0.8) p.u.	Normal load (1.0) p.u.	Load increased 20% (1.2) p.u.
Without STATCOM	-98.6815	-98.6747	-98.6701
	-0.3157 +	-0.3146 +	-0.3148 +
	8.074i	8.978i	9.799i
	-0.3157 -	-0.3146 -	-0.3148 -
	.0744i	8.9783i (0.035)	9.7990i (0.0321)
	(0.0391)	-1.7152	-1.7194
With STATCOM	-98.8545	-98.8533	-98.8525
	-0.3633 +	-0.3569 +	-0.3527 +
	5.069i	5.687i	6.243i
	-0.3633 -	-0.3569 -	-0.3527 -
	5.0690i (0.0715)	5.6875i (0.0626)	6.2438i (0.0564)
	-0.1139	-0.0881	-0.0719
With POD Controller	-98.8545	-98.8533	-98.8525
	-1.8166 +	-1.7847 +	-1.7637 +
	5.069i	5.687i	6.243i
	-1.8166 -	-1.7847 -	-1.7637 -
	5.0690i (0.337)	5.6875i (0.299)	6.2438i (0.272)
	-1.3681	-1.4079	-1.4333
	-0.1139	-0.0881	-0.0719

Table2: Eigen values with STATCOM POD controller for strong SMIB system.

- **Simulation results under different system conditions and various loading conditions**

It can be readily seen that the proposed controller performs better in terms of reduction of overshoot and settling time than system without STATCOM and with system with STATCOM only. This is consistent with the eigenvalues analysis results. Simulation results with variation in system- state, rotor angle (ϕ) of generator is only considered. The system responses are simulated using M-file program of MATLAB. Figures 3 to 6 show the combined system response without STATCOM, with STATCOM and with STATCOM POD controller at 1.0 pu, 0.8 pu and 1.2 pu line loading with 0.85 power factor of weaker (damping coefficient D=0) and stronger (damping coefficient D=4) SMIB power system. It can be observed from these figures that the STATCOM with coordinated POD controller can greatly improve the damping of the system and its stability under different line loading and system condition as mention above.

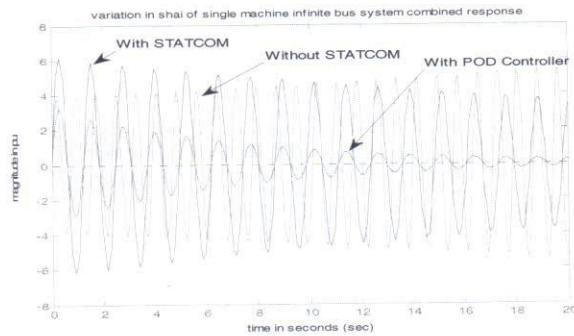


Fig. 3 Response at D=0, Load=0.8 p.u.

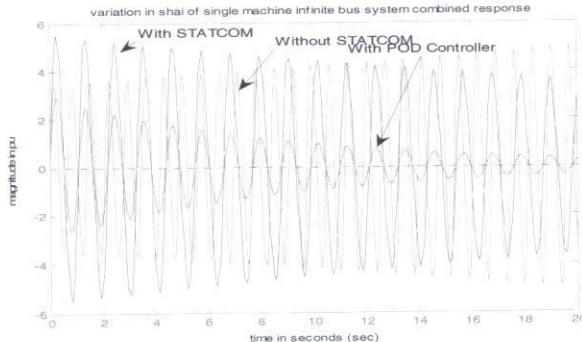


Fig. 4 Response at D=0, Load=1 p.u.

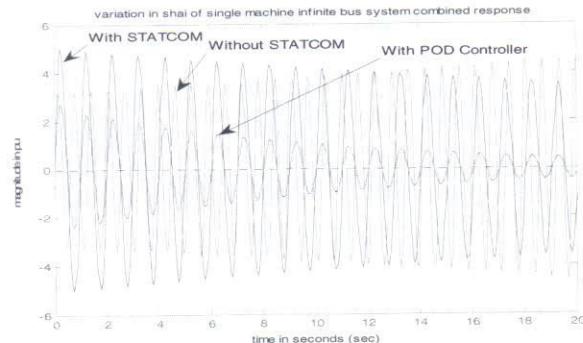


Fig. 5 Response at D=0, Load=1.2 p.u.

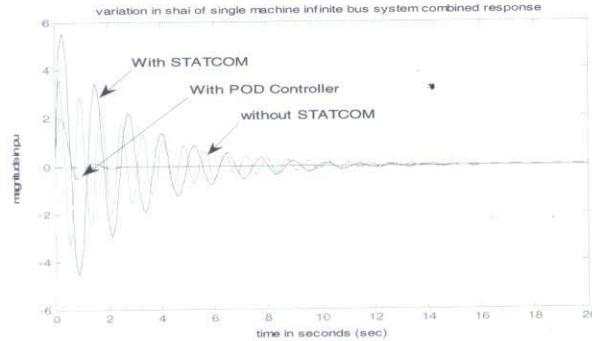


Fig. 6 Response at D=4, Load=0.8 p.u.

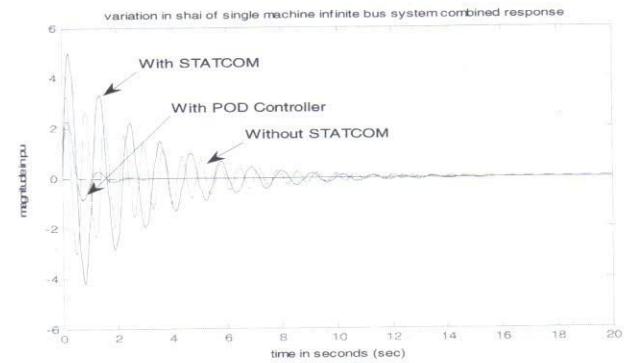


Fig. 7 Response at D=4, Load=1.0 p.u.

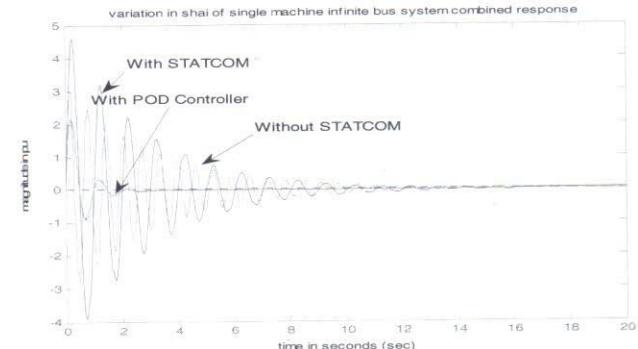


Fig. 8 Response at D=4, Load=1.2 p.u.

• Eigen- values and simulation results with variation in modulation index

modulation index Controller	m= 0.3 p.u.	m= 0.7 p.u.	m=1 p.u.
Without STATCOM	-98.6747	-98.6747	-98.6747
	-0.3146 + 8.9783i	-0.3146 + 8.9783i	-0.3146 + 8.9783i
	-0.3146 - 8.9783i (0.0345)	-0.3146 - 8.9783i (0.035)	-0.3146 - 8.9783i (0.035)
	-1.7152	-1.7152	-1.7152
With STATCOM	-98.8539	-98.8535	-98.8533
	-0.3711 + 5.8259i	-0.3652 + 5.7481i	-0.3569 + 5.6875i
	-0.3711 - 5.8259i (0.0636)	-0.3652 - 5.7481i (0.0634)	-0.3569 - 5.6875i (0.0626)
	-0.0073	-0.0414	-0.0881
	-1.4597	-1.4378	-1.4079
With POD Controller	-98.8539	-98.8535	-98.8533
	-1.8557 + 5.8259i	-1.8260 + 5.7481i	-1.7847 + 5.6875i
	-1.8557 - 5.8259i (0.305)	-1.8260 - 5.7481i (0.303)	-1.7847 - 5.6875i (0.299)

	-2.9194	-0.0827	-0.1762
	-0.0146	-2.8757	-2.8158

Table 3 Variation in modulation index of VSC with different system conditions

The variation in modulation index shows in the table-3 in which the study of eigenvalues is done. From this study it is find that as the modulation index increases stability of the system decreases. and the simulation results as shown in fig. 7,8 and 9.

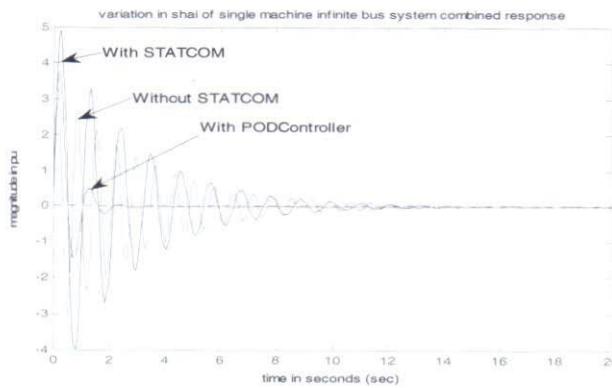


Fig. 9 Response at 1 p.u. load and D=4, m=0.3

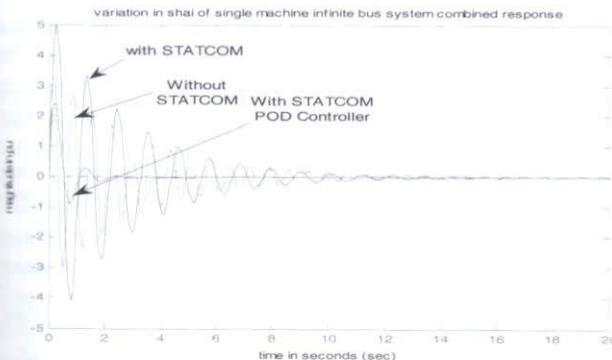


Fig. 10 Response at 1 p.u. load and D=4, m=0.7

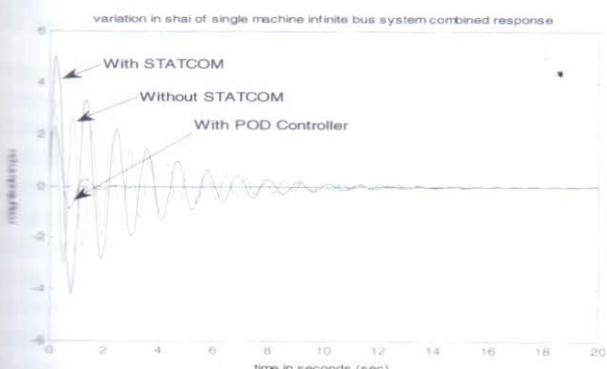


Fig. 11 Response at 1 p.u. load and D=4, m=1

- **Eigen- values and simulation results with variation in converter angle**

The variation in converter angle of STATCOM is another equally important factor for the study of performance POD controller using eigenvalue analysis. Under this section converters phase angle at 70 degree, 80 degree and 90 degree are taken respectively for the eigenvalue analysis and study the performance of POD controller. Table-4 shows the combined study of eigenvalues of variations in converter angle of STATCOM. From this table it can be observe that at lower value of the variation in converter phase angle is more effective than higher value of angle, The bolded row of this table represents the electromechanical mode eigenvalue and its damping ratio.

The comparative study from this table-3 and table-4 shows that variation in converter phase angle is more effective than variation in amplitude modulation index of converter; and the simulation results are shown in fig. 7 to 12. hence performance of proposed controller is more effective.

TABLE 4: VARIATION IN CONVERTER ANGLE WITH DIFFERENT SYSTEM CONDITIONS

Converte r angle Controll er	Angle=70 deg.	Angle =80 deg.	Angle = 90 deg.
Without STATC OM	-98.6747 -0.3146 + 8.9783i -0.3146 - 8.9783i (0.035) -1.7152	-98.6747 -0.3146 + 8.9783i -0.3146 - 8.9783i (0.035) -1.7152	-98.6747 -0.3146 + 8.9783i -0.3146 - 8.9783i (0.035) -1.7152
With STATC OM	-98.8608 -0.5042 + 5.6361i -0.5042 - 5.6361i (0.0891) -0.3214 -1.3164	-98.8566 -0.4473 + 5.6595i -0.4473 - 5.6595i (0.0788) -0.1923 -1.3559	-98.8533 -0.3569 + 5.6875i -0.3569 - 5.6875i (0.0626) -0.0881 -1.4079
With POD Controll er	-98.8608 -2.5212 + 5.6361i -2.5212 - 5.6361i (0.408) -0.6428 -2.6328	-98.8566 -2.2363 + 5.6595i -2.2363 - 5.6595i (0.367) -0.3846 -2.7117	-98.8533 -1.7847 + 5.6875i -1.7847 - 5.6875i (0.299) -0.1762 -2.8158

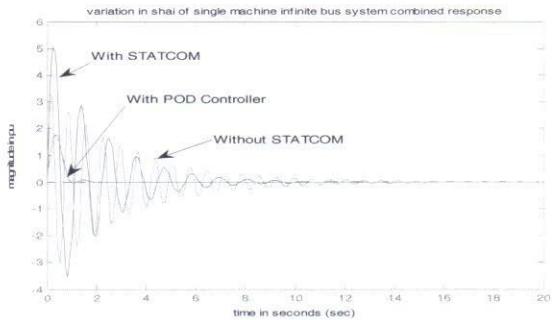


Fig .12: Response at 1 p.u. load and D=4, m=1, $\phi = 70$ deg

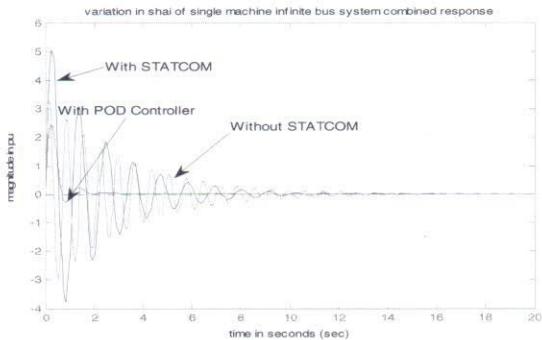


Fig .13: Response at 1 p.u. load and D=4, m=1, $\phi = 80$ deg

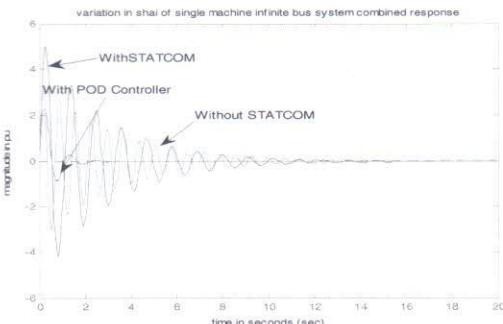


Fig .14: Response at 1 p.u. load and D=4, m=1, $\phi = 90$ deg

VI. CONCLUSIONS

Power oscillation damping and hence improving power system dynamic stability have been verified through eigen-value analysis and simulation results with different system conditions and under different line loading. The effectiveness

of proposed controller in damping low frequency EM mode of oscillations and hence improving the power system stability also have been verified through eigenvalue analysis and simulation results with the variation in modulating index of and phase angle of the converter.

APPENDIX

$$\begin{aligned}
 C_{DC} &= 1.0, \\
 E_q^/ &= 1.0, K_a = 10, H = 3 \text{ MJ / MVA}, \\
 M &= 2H, p_f = 0.85, T_a = 0.01 \text{ sec}; \\
 V_b &= 1.0, V_{DC} = 1.0, X_d = 1.0, X_d^/ = 0.3, \\
 X_q &= 0.6, T_{d0}^/ = 5.044 \text{ sec}, \\
 \delta &= 30 \text{ deg.} \\
 X_{SDT} &= 0.15 \\
 X_{tL} &= 0.3, X_{LB} = 0.3
 \end{aligned}$$

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